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**PIGOUVIAN TAX AND THE CONGESTION EXTERNALITY:  
A BENEFIT SIDE APPROACH**

by

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# **PIGOUVIAN TAX AND THE CONGESTION EXTERNALITY:**

## **A BENEFIT SIDE APPROACH**

**Kenneth S. Lyon and Dug Man Lee**

### **ABSTRACT**

A Pigouvian tax, in the form of an entrance fee or a toll, has been proposed to be an efficient resolution of a congestion externality. This proposition is founded on the theoretical basis of the profit maximization principle. We, however, have not found literature that examines a Pigouvian tax as a resolution of a congestion externality on the basis of utility maximization. In this sense, the aim of this paper is to show that a Pigouvian tax is also an adequate policy resolution of a congestion externality to attain Pareto optimality under utility maximization. Taking, for example, the open access freeway, we will not only identify both marginal private benefit and marginal social benefit, but also assess the divergence between marginal private benefit and marginal social benefit. As a consequence, we will investigate the price-taker individual's contribution to the congestion externality and measure, in the theoretical sense, how large an entrance fee or toll that should be charged to attain Pareto optimality. In particular, since an open access freeway has the characteristics of common property resource, we will prove that average social congestion cost is essentially equal to marginal private congestion cost in our modeling framework. Finally, we will show that the optimal value of trip derived in our model is the same as that generated on profit maximization approach.

*JEL* Classification: H21, H23

Key words: congestion externality, Pareto optimality, marginal private benefit, marginal social benefit

# PIGOUVIAN TAX AND THE CONGESTION EXTERNALITY:

## A BENEFIT SIDE APPROACH

### INTRODUCTION

Congestion externality has a long history as one of important subjects in economics. A.C. Pigou (1920) initially tackled this problem in his famous book, *The Economics of Welfare*, taking an example of the case of two roads. Knight (1923) also analyzed it in more detail and proved that if the owner of a superior road charges a toll for its use, this behavior will maximize the total product of both roads. For his arguments, he illustrated two alternative methods of analyzing the two-road problem, one is the cost side approach hinged on increasing costs, and the other is product side approach based on diminishing returns. Since Knight described his idea on congestion externality problem, follow-up literatures (Walters 1960, Samuelson 1974, Wizman 1974) have also suggested a Pigouvian tax, in the form of an entrance fee (rents) or a toll, as a resolution of a congestion externality. Their propositions are founded on the theoretical basis of the profit-maximizing principle. We, however, have not found literature that examines a Pigouvian tax as resolution of a congestion externality on the basis of utility maximization.

In this sense, the aim of this paper is to show that a Pigouvian tax is also an adequate policy resolution of a congestion externality to attain Pareto optimality under utility maximization. Taking, for example, the open access freeway, we will not only identify both marginal private benefit and marginal social benefit. As usual, an open-access freeway is accounted as a common property resource since it is nonexclusively owned, and we will also prove that average social congestion cost is essentially equal to marginal private congestion cost in our model. We next identify a

price-taker individual's utility maximization problem, and derive an expression for marginal private benefit. Following this we solve the Pareto optimality problem to identify marginal social benefit at the optimum. With these models, we will not only investigate the price-taker individual's contribution to the congestion externality, but also estimate, in the theoretical sense, how large an entrance fee or a toll should be charged to attain Pareto optimality.

#### PRICE-TAKER INDIVIDUAL PROBLEM.

A price-taker individual (denoted as superscript  $b$ ) is posited to maximize his utility. To set up the individual's utility maximization problem, we define the individual's utility function as  $u^b(c_1^b, c_2^b, y^b, h^b)$  with potential derivatives  $u_1^b, u_2^b, u_y^b > 0$  where  $c_1$  is a composite consumption good (money-numeraire good),  $c_2$  is a composite consumption good connected with leisure time,  $y$  is the number of trips per time period, and  $h$  is hours worked. The individual is constrained by both a budget and a time constraint. Under these conditions, the individual's utility maximization problem is as follows:

$$(1) \quad \text{Max } u^b(c_1^b, c_2^b, y^b, h^b)$$

Subject to

$$(2) \quad c_1^b + p_2 c_2^b + p_y y^b - w h^b - b = 0$$

$$(3) \quad t_2 c_2^b + t_y y^b + h^b - T = 0$$

$$(4) \quad t_y = g(y) = g(y^b + y^a) \quad g' > 0$$

$y^a, p_2, p_y, w, t_2$  and  $b$  are given.

where  $t_y$  is the time-price or time coefficient of a trip with  $g$  capturing the effects of congestion,  $p_y$  is the money price of a trip,  $w$  is the wage rate,  $b$  is non-labor income, the  $t_2$  is time price or time coefficient for the consumption of  $c_2$ , and  $T$  is total amount of time in the time period. Superscript  $a$  denotes a composition all individuals except individual  $b$ , and  $g$  is the index of congestion.

The Lagrangian function is

$$L = u^b(c_1^b, c_2^b, y^b, h^b) + \lambda_1(wh^b + b - c_1^b - p_2c_2^b - p_y y^b) + \lambda_2(T - t_2c_2^b - t_y y^b - h^b)$$

Some of the first order necessary conditions of this problem are

$$(5) \quad u_{c_1}^b(v^{b\oplus}) - \lambda_1^\oplus = 0$$

$$(6) \quad u_{c_2}^b(v^{b\oplus}) - \lambda_1^\oplus p_2 - \lambda_2^\oplus t_2 = 0$$

$$(7) \quad u_y^b(v^{b\oplus}) - \lambda_1^\oplus p_y - \lambda_2^\oplus [g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})] = 0$$

$$(8) \quad u_h^b(v^{b\oplus}) + \lambda_1^\oplus w - \lambda_2^\oplus = 0$$

where  $v^b$  is a vector containing  $c_1^b, c_2^b, y_1^b$ , and  $h^b$  as elements. The super  $\oplus$  indicates optimal values. We use  $c_1$  as the numeraire and use the marginal rate of substitution as the measure of value. We can derive the following expression for the value of a trip (See the Appendix for the derivation);

$$(9) \quad S_{c_1 y}^b(v^{b\oplus}) = [S_{c_1 h}^b(v^{b\oplus}) + w][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})] + p_y$$

where  $S$  denotes the marginal rate of substitution. In equation (9), we can observe that the term,  $[g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})]$ , denotes the marginal private time congestion cost (MPCC). This follows from the total private time congestion cost,  $t_y \cdot y^b$ . Also,

$S_{c_1 h}^b(v^{b\oplus})$  is the value at the margin of  $h$ , work time, in terms of  $c_1$ , and  $w$  is wage rate;

thus  $[S_{c_1 h}^b(v^{b\oplus}) + w]$  denotes the marginal value of  $h$  in terms of both time on the job

and the money income that is earned. Hence, we can interpret the term,

$[S_{c_1 h}^b(v^{b\oplus}) + w][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})]$ , as the value of the marginal private time congestion cost (*VMPCC*).

### PARETO OPTIMALITY PROBLEM

To identify society's Pareto optimality conditions in the face of congestion externality, we examine the following maximization problem. Individual  $b$ 's utility is maximized subject to given utility level for Individual  $a$ , who stands for everyone else<sup>1</sup>, and other constraints. The optimization problem for Pareto optimality is

$$(10) \quad \text{Max } u^b(c_1^b, c_2^b, y^b, h^b)$$

Subject to

$$(11) \quad t_2 c_2^b + t_y y^b + h^b - T^b = 0$$

$$(12) \quad u^a(c_1^a, c_2^a, y^a, h^a) - u^{a0} = 0$$

$$(13) \quad t_2 c_2^a + t_y y^a + h^a - T^a = 0$$

$$(14) \quad t_y = g(y) = g(y^b + y^a) \quad g' > 0$$

$$(15) \quad c_1^a + c_1^b - c_1 = 0$$

$$(16) \quad c_2^a + c_2^b - c_2 = 0$$

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<sup>1</sup> We could use the sum of everyone else, which would be technically more appropriate; however, this would not have a substantive effect.



$$(17) \quad y^b + y^a - y = 0$$

$$(18) \quad h^a + h^b - h = 0$$

$$(19) \quad H(c_1, c_2, y, h) = 0 \quad H_{c_1}, H_{c_2}, H_y < 0 \quad \text{and} \quad H_h > 0$$

where  $H$  is the production possibility function. For this optimization problem, the Lagrangian function is

$$L = u^b(c_1^b, c_2^b, y^b, h^b) + \lambda_1[T^b - t_2 c_2^b - g(y)y^b - h^b] + \lambda_2[u^{a0} - u^a(c_1^a, c_2^a, y^a, h^a) \\ + \lambda_3[T^a - t_2 c_2^a - g(y)y^a - h^a] + \lambda_4 H(c_1^a + c_1^b, c_2^a + c_2^b, y^a + y^b, h^a + h^b)$$

Some of the first order necessary conditions are

$$(20) \quad u_{c_1}^b(v^{b*}) + \lambda_4^* H_{c_1}(v^*) = 0$$

$$(21) \quad u_{c_2}^b(v^{b*}) - \lambda_1^* \lambda_2^* + \lambda_4^* H_{c_2}(v^*) = 0$$

$$(22) \quad u_y^b(v^{b*}) - \lambda_1^* [g'(y^*)y^{b*} + g(y^*)] - \lambda_3^* g'(y^*)y^{a*} + \lambda_4^* H_y(v^*) = 0$$

$$(23) \quad u_h^b(v^{b*}) - \lambda_1^* + \lambda_4^* H_h(v^*) = 0$$

$$(24) \quad -\lambda_2^* u_{c_1}^a(v^{a*}) + \lambda_4^* H_{c_1}(v^*) = 0$$

$$(25) \quad -\lambda_2^* u_{c_2}^a(v^{a*}) - \lambda_3^* t_2 + \lambda_4^* H_{c_2}(v^*) = 0$$

$$(26) \quad -\lambda_2^* u_y^a(v^{a*}) - \lambda_1^* g'(y^*)y^{b*} - \lambda_3^* [g'(y^*)y^{a*} + g(y^*)] + \lambda_4^* H_y(v^*) = 0$$

$$(27) \quad -\lambda_2^* u_h^a(v^{a*}) - \lambda_3^* + \lambda_4^* H_h(v^*) = 0$$

where  $v^a$  is a vector containing  $c_1^a, c_2^a, y^a$ , and  $h^a$  as elements, and also  $v$  is a vector

having  $c_1, c_2, y$ , and  $h$  as elements. The super asterisk implies optimal values. Note that

$c_1$  is the money numeraire, and if we have a commodity with no money price but a time

price, we could make it the time-numeraire commodity. Manipulating the first order necessary conditions of this problem, the marginal social benefit for  $y^b$  is

$$(28) \quad S_{c1y}^b(v^{b*}) = [S_{c1h}^b(v^{b*}) + w^*][g'(y^*)y^{b*} + g(y^*)] + [S_{c1h}^a(v^{a*}) + w^*]g'(y^*)y^{a*} + ROT_{c1y}(v^*)^2$$

and the marginal social benefit for  $y^a$  would be

$$(29) \quad S_{c1y}^a(v^{a*}) = [S_{c1h}^a(v^{a*}) + w^*][g'(y^*)y^{a*} + g(y^*)] + [S_{c1h}^b(v^{b*}) + w^*]g'(y^*)y^{b*} + ROT_{c1y}(v^*)$$

where  $S$  is marginal rate of substitution,  $w^*$  is the wage rate, and  $ROT^*$  is the rate of transformation at the optimum (See the Appendix for the derivation).

We now examine equation (28) in detail. The first term,  $[S_{c1h}^b(v^{b*}) + w^*][g'(y^*)y^{b*} + g(y^*)]$ , denotes the internal time cost to Individual  $b$  of consuming freeway services. It is the value of the marginal private time congestion cost (*VMPPC*) as described in connection with equation (9). The second term,  $[S_{c1h}^a(v^{a*}) + w^*]g'(y^*)y^{a*}$ , is the external time cost, which is the value of the marginal time cost to Individual  $a$  incurred because of  $b$ 's freeway usage. If we compare equation (9) with equation (28), we can easily see that there exists divergence between the marginal private benefit and the marginal social benefit at a given quantity of  $y^b$ . This divergence is the external time cost. The

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<sup>2</sup> We know that  $ROT_{c1y}(v^*) = H_y(v^*)/H_{c1}(v^*)$  along production possibility curve, and to attain Pareto optimality,  $ROT_{c1y}(v^*)$  should be equal to  $S_{c1y}(v^*)$ , which is the value of trips per time period in terms of  $c_1$ . Thus we can identify that  $ROT_{c1y}(v^*) = p_y$  at Pareto optimality. In this we assume price-taker firms.

optimal Pigouvian tax would, therefore, be  $[S_{clh}^a(v^{a*}) + w^*]g'(y^*)y^{a*}$ . This tax would give

Individual  $b$  the incentive to adjust his/her usage of freeway services to the efficient level.

This Pigouvian tax coincides with the profit or wealth maximizing toll as shown below.

Using equations (9) and (28), we can draw Figure 1 as the mirror image of the one depicted in literature that studied congestion externality on the basis of profit maximization. Figure 1 not only identifies the extent of Individual  $b$ 's over-utilization of the freeway, but also depicts the Pigouvian tax that could be levied to attain Pareto optimality. In the Figure, the distance of  $y^{b\oplus} - y^{b*}$  denotes the over-utilization of freeway service of the price-taker individual, and the length of  $AB$  gives the Pigouvian tax. This is  $[S_{clh}^a(v^{a*}) + w^*]g'(y^*)y^{a*}$  denoted in the equation (28)<sup>3</sup>. In Figure 1,  $\hat{v}^b$  is a

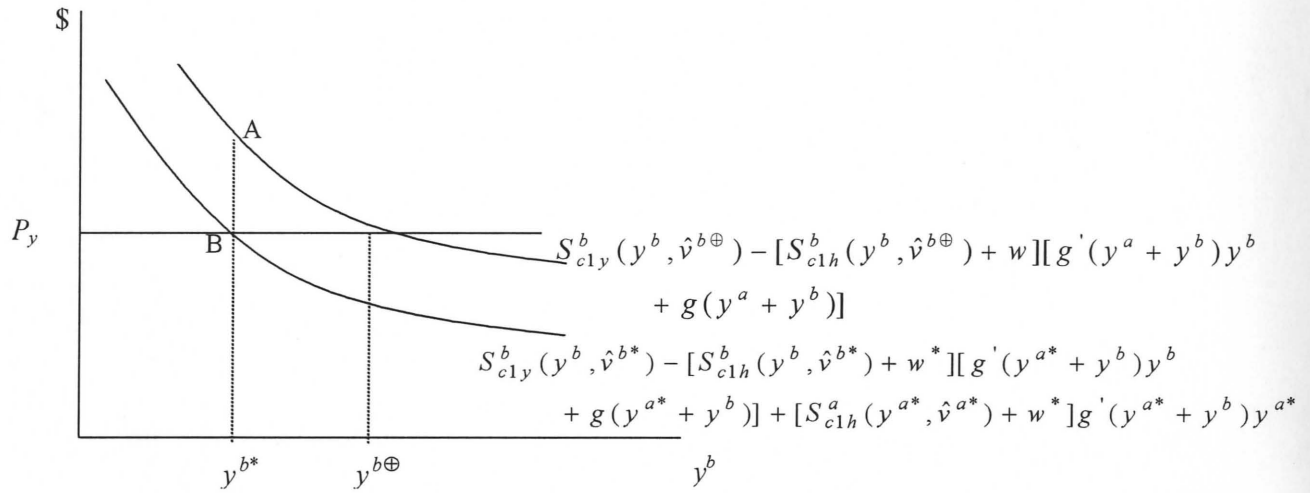


Figure 1

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Also we let  $w^* = H_h(v^*)/H_{cl}(v^*)$ , which is the value at the marginal product of labor in the production of  $c_1$ , our numeraire good.

vector containing  $c_1^b, c_2^b, y_1^b$  as elements, and  $\hat{v}^a$  is also a vector having  $c_1^a, c_2^a, y^a$  as elements.

We now discuss the proposition that the average social congestion cost is essentially equal to the marginal private congestion cost. The social time congestion cost is defined as  $g(y)y$ ; thus, the average social congestion cost (*ASCC*) is  $g(y)$  and the marginal social time congestion cost (*MSCC*) is  $g'(y)y + g(y)$ . To simplify the discussion, we assume that Individual  $b$  is a typical individual so that it is reasonable to assume that  $S_{clh}^b(v^{b*}) = S_{clh}^a(v^{a*})$ . In addition, we assume that these rates of substitution are constant over the range of the variables discussed. Let this constant value be given by  $S_{clh}^*$ . Using equation (28) the value of the marginal social time congestion cost at the optimum is

$$VMSCC^* = [S_{clh}^* + w^*][g'(y^*)y^* + g(y^*)]$$

and using equation (9) the value of the marginal value of private time congestion cost at Individual  $b$ 's optimum is

$$VMPCC^\oplus = [S_{clh}^* + w][g'(y^{b\oplus} + y^a)y^{b\oplus} + g(y^{b\oplus} + y^a)]$$

To reach the conclusion that the value of the marginal private time congestion cost is essentially equal to the value of the average social time congestion cost, we assume that  $g'$  does not change rapidly so that  $g'(y^{b*} + y^{a*})$  and  $g'(y^{b\oplus} + y^a)$  are approximately

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<sup>3</sup> If the index of congestion,  $g$ , and  $S_{clh}^a(v^{a*})$  were known, we could estimate  $g'(y^*)$  and thereby measure the term,  $[S_{clh}^a(v^{a*}) + w^*]g'(y^*)y^{a*}$ . However, estimation of the congestion cost is beyond the scope of this paper.

equal. Because  $y^b$  is small compared with  $y^a$ , which is for everyone else,  $g'(y^{b\oplus} + y^a)y^{b\oplus}$  is small compared with  $g'(y^{b*} + y^{a*})y^*$ . When  $y^{b\oplus}$  is imperceptible  $g'(y^{b\oplus} + y^a)y^{b\oplus}$  is essentially equal to zero. Another justification for ignoring this term is that it is Individual  $b$ 's own marginal effect on congestion and the individual can be expected to ignore this effect. When I enter a congested area it is not my effect that I notice, rather it is the effect of others. With this product essentially equal to zero the value of marginal private time congestion cost is essentially equal to the value of average social time congestion cost. This is a very common conclusion in this literature; hence, one view of this paragraph is that this is a sufficient set of assumptions to generate this conclusion.

We now explore how these results on utility maximization approach are related with those on the profit maximization approach. Frank Knight proved that private ownership of Pigou's narrow road would lead to a maximization of the value of the road and a social optimum. Our results yield the same conclusion for the same reason. The right most curve in Figure 1 is Individual  $b$ 's demand function for trips in inverse form. Because we seek information about a toll that is charged in addition to  $p_y$ , we are interested only in the portion of the curve above of  $p_y$ . The sum of these curves over all individuals yields the aggregate demand function for trips and is the average social value curve; hence the associated marginal curve is the marginal social value curve. We assume, as is usual, that the marginal cost of collecting the toll is zero; hence the toll that would maximize total profits (wealth) to the toll collector is where marginal revenue equals zero. This is where marginal social value equal zero, but since we subtracted out  $p_y$  in Figure 1,

this is where marginal social value equals  $p_y$ ; hence, this approach yields the optimal value depicted in Figure 1. This optimal value of trip will be the same as that generated on profit or wealth maximization approach.

## SUMMARY

In this paper we have shown that a Pigouvian tax is an adequate resolution of congestion externality to attain Pareto optimality using utility maximization. For this objective, taking an open access freeway as an example, we not only derived both marginal private benefit and marginal social benefit, but also assessed the divergence between marginal private benefit and marginal social benefit. As a result, we identified that the amount of a Pigouvian tax should be the same amount as the external time cost, which is the value of the marginal time cost to Individual  $a$  incurred by Individual  $b$  through freeway congestion. This Pigouvian tax coincides with the profit or wealth maximizing toll suggested by literature on the basis of profit maximization. In addition, because an open access freeway is accounted as common property resource, we proved that average social congestion cost is essentially equal to marginal private congestion cost in our model.

## APPENDIX

Derivation of equation (9)

We get from equation (5)

$$u_{c1}^b(v^{b\oplus}) = \lambda_1^\oplus$$

From equation (8)

$$u_h^b(v^{b\oplus}) + u_{c1}^b(v^{b\oplus})w = \lambda_2^\oplus$$

Combining these two results with equation (7),

$$u_y^b(v^{b\oplus}) - u_{c1}^b(v^{b\oplus})p_y - [u_h^b(v^{b\oplus}) + u_{c1}^b(v^{b\oplus})w][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})] = 0$$

$$u_y^b(v^{b\oplus}) - u_{c1}^b(v^{b\oplus})[p_y + \{u_h^b(v^{b\oplus})/u_{c1}^b(v^{b\oplus}) + w\}][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})] = 0$$

$$u_y^b(v^{b\oplus})/u_{c1}^b(v^{b\oplus}) = p_y + [u_h^b(v^{b\oplus})/u_{c1}^b(v^{b\oplus}) + w][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})]$$

From this manipulation, we can derive equation (9)

$$S_{c1y}^b(v^{b\oplus}) = [S_{c1h}^b(v^{b\oplus}) + w][g'(y^a + y^{b\oplus})y^{b\oplus} + g(y^a + y^{b\oplus})] + p_y$$

Derivation of equation (28)

We can get from equation (20)

$$\lambda_4^* = -u_{c1}^b(v^{b*})/H_{c1}(v^*)$$

From equation (23)

$$\lambda_1^* = u_h^b(v^{b*}) - [H_h(v^*)/H_{c1}(v^*)]u_{c1}^b(v^{b*})$$

From equation (22)

$$u_y^b(v^{b*}) - [u_h^b(v^{b*}) - \{H_h(v^*)/H_{c1}(v^*)\}u_{c1}^b(v^{b*})][g'(y^*)y^{b*} + g(y^*)]$$

$$-\lambda_3^* g'(y^*) y^{a*} - [H_y(v^*) / H_{cl}(v^*)] u_{cl}^b(v^{b*}) = 0 \quad (A-1)$$

From equation (24)

$$-\lambda_2^* u_{cl}^a(v^{a*}) - [u_{cl}^b(v^{b*}) / H_{cl}(v^*)] H_{cl}(v^*) = 0$$

$$\lambda_2^* = -u_{cl}^b(v^{b*}) / u_{cl}^a(v^{a*})$$

From equation (27)

$$[u_{cl}^b(v^{b*}) / u_{cl}^a(v^{a*})] u_h^a(v^{a*}) - \lambda_3^* - [u_{cl}^b(v^{b*}) / H_{cl}(v^*)] H_h(v^*) = 0,$$

$$\lambda_3^* = [u_{cl}^b(v^{b*}) / u_{cl}^a(v^{a*})] u_h^a(v^{a*}) - [u_{cl}^b(v^{b*}) / H_{cl}(v^*)] H_h(v^*)$$

With these results, equation (A-1) can be rearranged into,

$$\begin{aligned} & u_y^b(v^{b*}) - [u_h^b(v^{b*}) - \{H_h(v^*) / H_{cl}(v^*)\} u_{cl}^b(v^{b*})] [g'(y^*) y^{b*} + g(y^*)] \\ & - u_{cl}^b(v^{b*}) [u_h^a(v^{a*}) / u_{cl}^a(v^{a*}) - H_h(v^*) / H_{cl}(v^*)] g'(y^*) y^{a*} - [H_y(v^*) / H_{cl}(v^*)] u_{cl}^b(v^{b*}) = 0 \\ & u_y^b(v^{b*}) / u_{cl}^b(v^{b*}) = [u_h^b(v^{b*}) / u_{cl}^b(v^{b*}) - H_h(v^*) / H_{cl}(v^*)] [g'(y^*) y^{b*} + g(y^*)] \\ & + [u_h^a(v^{a*}) / u_{cl}^a(v^{a*}) - H_h(v^*) / H_{cl}(v^*)] g'(y^*) y^{a*} + H_y(v^*) / H_{cl}(v^*) \end{aligned}$$

This completes the derivation of equation (28),

$$\begin{aligned} S_{cly}^b(v^{b*}) &= [S_{clh}^b(v^{b*}) + w^*] [g'(y^*) y^{b*} + g(y^*)] + [S_{clh}^a(v^{a*}) + w^*] g'(y^*) y^{a*} \\ &+ ROT_{cly}(v^*) \end{aligned}$$

Equation (29) can be also derived using the same method.



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## Pigouvian Tax and the Congestion Externality: A Benefit Side Approach

### ABSTRACT

A Pigouvian tax, in the form of entrance fee or a toll, has been proposed to be an efficient resolution of a congestion externality. This proposition is founded on the theoretical basis of profit maximization principle. We, however, have not found literature that examines a Pigouvian tax as a resolution of a congestion externality on the basis of utility maximization. In this sense, the aim of this paper is to show that a Pigouvian tax is also an adequate policy resolution of a congestion externality to attain Pareto optimality under utility maximization. Taking, for example, the open access freeway, we will not only identify both marginal private benefit and marginal social benefit, but also assess the divergence between marginal private benefit and marginal social benefit. As a consequence, we will investigate the price-taker individual's contribution to the congestion externality and measure, in the theoretical sense, how large an entrance fee or toll that should be charged to attain Pareto optimality. In particular, since an open access freeway has the characteristics of common property resource we will prove that average social congestion cost is essentially equal to marginal private congestion cost in our modeling framework. Finally, we will show that the optimal value of trip derived in our model is the same as that generated on profit maximization approach.

## INTRODUCTION

Congestion externality has a long history as one of important subjects in economics. A.C. Pigou (1920) initially tackled this problem in his famous book, *The Economics of Welfare*, taking an example of the case of two roads. Knight (1923) also analyzed it in more detail and proved that if the owner of a superior road charges a toll for its use, this behavior will maximize the total product of both roads. For his arguments, he illustrated two alternative methods of analyzing the two road problem, one is the cost side approach hinged on increasing costs, and the other is product side approach based on diminishing returns. Since Knight described his idea on congestion externality problem, follow-up literatures (Walters 1960, Samuelson 1974, Weizman 1974) have also suggested a Pigouvian tax, in the form of an entrance fee (rents) or a toll, as a resolution of a congestion externality. Their propositions are founded on the theoretical basis of the profit maximizing principle. We, however, have not found literature that examines a Pigouvian tax as resolution of a congestion externality on the basis of utility maximization.

In this sense, the aim of this paper is to show that a Pigouvian tax is also an adequate policy resolution of a congestion externality to attain Pareto optimality under utility maximization. Taking, for example, the open access freeway, we will not only identify both marginal private benefit and marginal social benefit, but also assess the difference between marginal private benefit and marginal social benefit. As usual, an open-access freeway is accounted as a common property resource since it is non-exclusively owned, and we will also prove that average social congestion cost is essentially equal to marginal private congestion cost in our model. We next identify a